Instability of The Standard Model by Squeezing Many Top Quarks

Yang Bai

Theoretical Physics Department, Fermilab

with Christopher T. Hill, working in progress

July 11, 2008, Santa Fe

Outline

Explore New Physics “Beyond” the Standard Model

- Question: Can the standard model tell us something beyond the standard model?
- Answer: **Yes**. For example:
  - the running of the gauge couplings strongly suggests a grand unified theory;
  - Need new physics to explain the dark matter;
  - the “Hierarchy Problem” suggests new *TeV* physics to be tested at LHC.
  - ......

Explore New Physics “Within” the Standard Model

- Question: Any other new physics can the standard model tell us by itself (without extending the field content)?
- Answer: **Yes**, if we study a many-body system composed by the elementary particles in the standard model.

How about new bound states?

For example, construct a bound state by using many top and anti-top quarks.

Definition of a bound state:

- Denote the mass of a system with n tops plus anti-tops as M_n.
- $M_n < M_{n-1} + m_t$ removing a single top quark
- $M_n < M_{n-1} + m_b$ a weak decay of a single internal top quark
- $M_n < M_{n-2}$ internal annihilation of a pair tops
- This system can only decay through multibody processes and the lifetime becomes long

New bound states of many top and anti-top quarks

$$\mathcal{L} = m_t \bar{t} t + \frac{g_t}{\sqrt{2}} h \bar{t} t + \frac{1}{2} m_h^2 h^2 + h.c. + \cdots$$

The attractive Yukawa potential between the two top (anti-top) quarks by exchanging a Higgs boson:

$$V(r) = -\frac{g_t^2/2}{4\pi r} e^{-m_h r}$$

If all top quarks are inside a sphere with the radius $r \leq R \ll 1/m_h$:

$$V(r) = -\frac{g_t^2/2}{4\pi r}$$

The simplest case: $t\,\bar{t}$ bound state

In the nonrelativistic limit, the Hamiltonian for the $t\,\bar{t}$ is

$$H = 2\,m_t + \frac{p^2}{2\,\bar{m}_t} - \frac{g_t^2}{8\pi r} \qquad \bar{m}_t = \frac{1}{2}m_t$$

Substitute $p = 1/r_B$ and $r = r_B$ and minimize the Hamiltonian w.r.t r_B.

$$r_B = \frac{16\pi}{g_t^2 m_t} \qquad M_{T_2} = 2m_t - \frac{g_t^4}{256\pi^2}m_t$$

Since $r_B > 1/m_h$, we should include the $e^{-m_h\,r}$ in the potential. The binding energy is highly suppressed. No stable toponium.

Existing Studies: T-ball [Froggatt, Nielsen and et al.]

They considered a $6\,t\; + \;6\bar{t}$ system, which occupy the $1S$ state of a Bohr atom. In the nonrelativistic limit, the Hamiltonian for N top quarks is

$$H = N\, m_t + \frac{p^2}{2\,\bar{m}_t} - \frac{\eta\, g_t^2}{8\pi r} \qquad \bar{m}_t = \frac{N-1}{N} m_t$$

Substitute $p = 1/r_B$ and $r = r_B$, minimize the Hamiltonian w.r.t r_B.

$$M_N = N\, m_t - (N-1)^3 \frac{\eta^2\, g_t^4}{256\pi^2} m_t$$

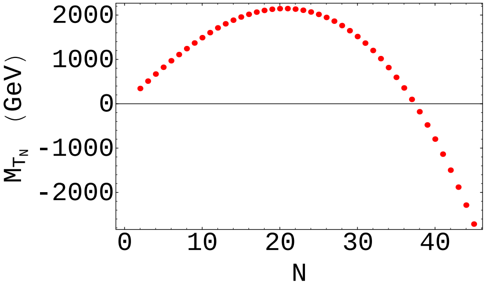
For $\eta \approx 2$ and $g_t \approx 1$, we have

$$M_{T_{12}} \approx 1710 \text{ GeV} \qquad M_{T_{11}} \approx 1630 \text{ GeV} \qquad M_{T_{10}} \approx 1530 \text{ GeV}$$

Extend the T-ball study

Consider 2S and 2P states. These allow additional 48 states.

$$M_{T_N} = N\, m_t - \frac{12(N-1)^3}{N}\frac{\eta^2 g_H^4}{256\pi^2} m_t - \frac{(N-12)(N-1)^3}{N}\frac{\eta^2 g_H^4}{4\times 256\pi^2} m_t$$



- For $22 < N < 37$, stable boundstates (Some are colored and good for LHC. Some are colorless and may be dark matter).
- For $38 \leq N$, the vacuum is unstable. We need to restudy the SM vacuum.

Nonrelativistic .vs. Relativistic

However, the momentum of the 1S state is

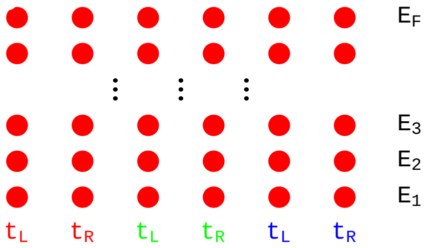
$$p = \frac{\eta(N-1)^2 g_H^2 m_t}{8\pi N} \sim 4\, m_t$$

Nonrelativistic approximation breaks down.

We need to consider relativistic limit.

Relativistic System and Fermi Sphere

- Now consider a relativistic assemblage of N top quarks (no anti-tops) that are squeezed into a small volume $\sim R^3$.
- The top mass and Higgs potential effects are negligible in the limit $R \ll 1/m_t$.
- We use the Fermi-Dirac statistics to analyze this system.
- First study the case that all states inside the Fermi sphere are occupied (State I).



The Kinetic Energy

The total number of top quarks is:

$$N = 2\,N_c\,V \int_0^{k_F} \frac{d^3k}{(2\pi)^3} = \frac{|k_F|^3\,N_c\,V}{3\,\pi^2} = \frac{4\,|k_F|^3\,R^3\,N_c}{9\,\pi}$$

This defines the Fermi momentum

$$k_F = \left[\frac{9\,\pi\,N}{4\,N_c}\right]^{1/3} /R$$

The resulting kinetic energy of the system is

$$\begin{aligned} K &= 2\,N_c\,V \int_0^{k_F} \frac{d^3k}{(2\pi)^3} \sqrt{|\vec{k}|^2 + m_t^2} \\ &\approx c\,\frac{N^{4/3}}{R}\,, \qquad\qquad c = \frac{3^{5/3}\,\pi^{1/3}}{4^{4/3}\,N_c^{1/3}}\,. \end{aligned}$$

Self Energy (State I)

Subtleties:

- The Higgs-exchange interaction of top quarks needs to flip helicity,
$$t_{L,R}(\vec{p}) \to t_{R,L}(\vec{p}) + h(\vec{0})$$
and all states inside the Fermi surface have already been filled.
- A t_L particle inside the Fermi surface can only go to a t_R state outside the Fermi surface and lead a large momentum for the Higgs boson propagator. Pauli Blocking
- Therefore, only top quarks on the Fermi surface can coherently interact with each other by Higgs exchange.

Self Energy (State I)

The total number of top quarks on the Fermi surface is

$$N_s = 6\,\pi^{1/3}\,N^{2/3}$$

The self-energy is

$$V_{self} = -\frac{9\,\eta^2\,g_t^4}{8\,\pi^{1/3}}\,\frac{N^{4/3}}{R}$$

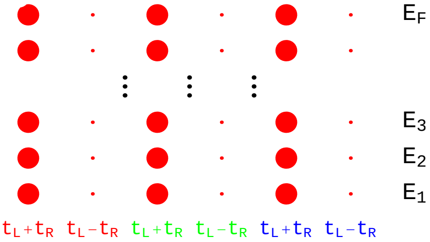
The total energy is

$$E = \left(\frac{3^{4/3}\,\pi^{1/3}}{4^{4/3}} - \frac{9\,\eta^2\,g_t^4}{8\,\pi^{1/3}}\right)\frac{N^{4/3}}{R} \sim -\frac{N^{4/3}}{R}$$

The system is unstable since $g_t^2 \approx 1.0 > g_{t_c}^2 \approx 0.57$ (The Chandrasekhar Limit analogous to the White Dwarf Star). Hence the current many top quark system will collapse.

Self Energy (State II)

Halfly occupied Fermi Sphere



no Pauli blocking

$$
\begin{aligned}
|\pm,\vec{p}\rangle &\equiv \frac{1}{\sqrt{2}}\left(|t_L,\vec{p}\rangle \pm |t_R,\vec{p}\rangle\right) \\
|\pm\rangle &\rightarrow |\pm\rangle + h(\vec{0})
\end{aligned}
$$

all top quarks inside the Fermi surface contribute to the self-energy.

Self Energy (State II)

The self energy is thus proportional to N^2 and negative:

$$V_{self} = -c' \frac{g_t^4 N^2}{R}, \qquad c' = \frac{3}{80\,\pi}$$

The total energy is

$$E = c\,\frac{N^{4/3}}{R} - c'\,\frac{g_t^4\,N^2}{R}$$

This system is unstable once N exceeds a critical value,

$$N_{crit} = \frac{1}{g_t^6}\left(\frac{c}{c'}\right)^{3/2} = \frac{5\,2^{3/4}\sqrt{15}\,\pi^2}{g_t^6} \approx 321$$

Vacuum Instability

Options

- No vacuum instability $\rightarrow$ no Higgs $\rightarrow$ dynamical symmetry breaking?
- We are living in a metastable vacuum. But the tunneling time to the real vaccum is longer than the age of universe.
- There is no short-distance repulsive core to prevent the system from collapsing. This may lead to a black-hole and another way of producing black-holes at the LHC.
- A feature of this result is classical scale invariant.
- The Higgs field may be heavy and the weak scale is emergent from multi-top condensation.
- $\cdots\cdots$

Relativistic Bound States

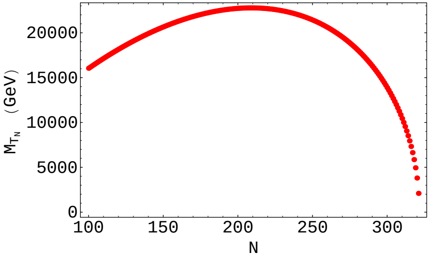
Need to introduce a scale, m_t. Consider the next leading term in the kinetic energy.

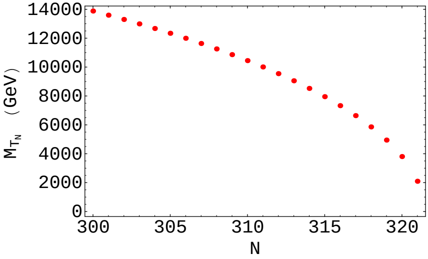
$$\begin{aligned} K &= N_c\, V \int_0^{k_F} \frac{d^3k}{(2\pi)^3} \sqrt{\vec{k}^2 + m_t^2} \approx N_c\, V \int_0^{k_F} \frac{d^3k}{(2\pi)^3} \left(k + \frac{m_t^2}{k}\right) \\ &= c_1 \frac{N^{4/3}}{R} + c_2\, m_t^2\, N^{2/3}\, R \qquad c_1 = \frac{3^{4/3}\,\pi^{1/3}}{2^{7/3}} \quad c_2 = \frac{3^{2/3}}{2\,\pi^{1/3}} \end{aligned}$$

The total energy

$$M_{T_N} = c_1 \frac{N^{4/3}}{R} + c_2\, m_t^2\, N^{2/3}\, R - c' \frac{g_t^4\, N^2}{R}$$

Relativistic Bound States





The lightest one is at $N = 321$. $M_{T_{321}} \approx 2$ TeV. The radius of the bound state is $R \sim 1/\text{TeV}$.

Conclusions

- Certain assemblage of many top quarks are unstable against collapse due to coherent Higgs boson potential energy. This implies a vacuum instability of the SM, but without a scale.

- There may exist a many-tops boundstate around the TeV scale. It deserves more efforts to study its collider signatures and cosmological consequences.